DYNAMICS AND EVOLUTIONARY STATUS OF THE YOUNG TRIPLE STELLAR SYSTEM, TY CRA

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Abstract. The young star TY CrA was known as a close eclipsing binary, which surprisingly appeared circularized but not synchronized. Recently, a third companion orbiting the close pair was detected. The dynamics of this triple system is here investigated. We show that according to pure 3-body dynamics, the TY CrA central binary should be subject to periodic changes related to the so-called Kozai mechanism in cometary dynamics, which should lead to a rapid collision. Adding tidal effects within that binary preserves its stability. We also show that thanks to the combination of tidal effects and 3-body dynamics, the rotation axes of the components of the central binary may by locked in a particular position (within the orbital plane) which might explain the apparent non-synchronism of the binary. Such a situation would not be stable if the central binary was alone, and if the system was older than $\sim 10^7$ yrs.

1. Introduction

The Herbig star TY CrA has been known as a short period eclipsing, spectroscopic binary [13, 17]. The identification of the radial velocity curves of both components allowed to determine their masses ($3.0\,M_\odot$ and $1.6M_\odot$), orbital characteristics, and rotation velocities [6]. The binary is almost tidally circularized ($e \simeq 0.02$), but the $3.0\,M_\odot$ primary appears surprisingly sub-synchronous [6]. As pointed out by Casey et al. [4], this cannot be explained straightforwardly.

Recently a third component orbiting the original binary system was detected [5, 7]. We recently presented [7] tentative fits of the orbital motion of the tertiary around the center of mass of the binary system, based on heliocentric velocity measurements. All solutions present in fact remarkable

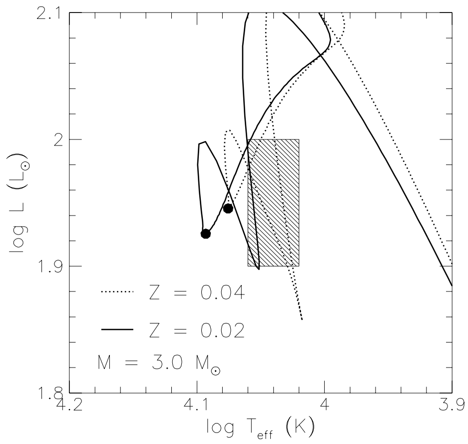

Figure 1. Hertzsprung-Russell diagram showing computed evolution tracks for $M = 3\,M_\odot$ and $Z = 0.02$ (solid line) or 0.04 (dashed line), as well as the observed position of the first TY CrA component taking account of the observational uncertainties.

common features (see [7]), in particular a high eccentricity ($e' \simeq 0.5$), a tertiary mass $m_3 \simeq 1.2 - 1.4\,M_\odot$, and an inclination of at least 70° with respect to the orbital plane of the eclipsing binary. Such an unusual configuration for a triple system is not surprising and is observed in other hierarchical multiple systems as well [9, 18].

2. Evolutionary status of TY CrA triple system

Although the TY CrA system is thought to be young, its age is controversial. The $3.0\,M_\odot$ primary has a total luminosity of $\sim 90L_\odot$ and effective temperature of 11 000K (see [17]). The luminosity of the whole system is dominated by the first component. The location of this component in the Hertzprung-Russell diagram normally allows to determine its total mass m_1 and age t, by comparison with computed evolutionary tracks. However, such theoretical tracks also depends on the metallicity Z of the modeled stars. As TY CrA consists in a young system, it appears reasonable to assume that its metallicity is quite similar or larger than the solar one, namely $Z \geq 0.02$. In Fig. 1, we show $3M_\odot$ theoretical tracks (corresponding to $Z = 0.02$ and 0.04) together with the observational box for TY CrA.

We note that the tracks penetrates the box twice (whatever Z), once before the zero-age main sequence is reached and again during the main sequence. This leads to two very different age predictions for the TY CrA system: one very small ($t_{\rm PMS} \simeq 3\,10^6$ yr or $3.5\,10^6$ yr for $Z = 0.02$ or 0.04), and another considerably larger ($t_{\rm MS} \simeq 1.5\,10^8$ yr or $\simeq 10^8$ yr for the same Z, respectively).

3. Three body dynamics

Although several non-planar triple stellar systems have been identified, it appears interesting to investigate the dynamics of such an unusual stellar

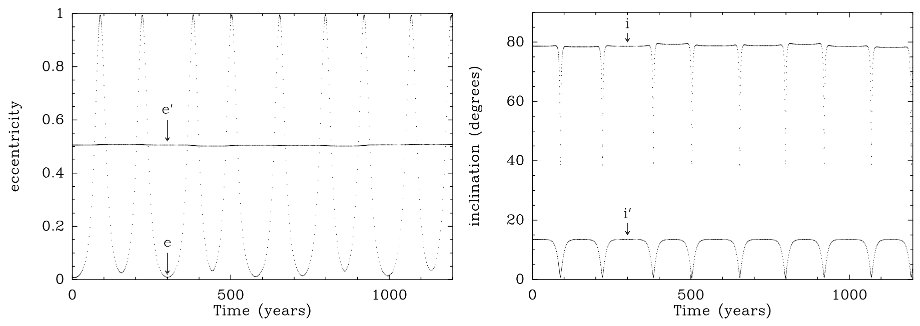

Figure 2. Evolution of the eccentricities and inclinations of the close binary (e, i) and the tertiary orbit (e', i') as a function of time for 1 200 years under the effects of pure 3-body dynamics. A quasi-periodic behavior (confirmed by integration over a longer time scale) is clearly detected; the close binary becomes very eccentric every $\sim$150 years. Conversely, tertiary orbit is very stable. Only very small variations are recorded.

system. Due to three body motion, the two considered orbits (binary system and tertiary) interact and may evolve slowly. Also the question of the stability of the system might be addressed. Finally, this could give clues for understanding the problem of the non-synchronism of the binary.

The interaction between the three components of the TY CrA system can be separated into three body interactions and tidal effects. We will see below that tidal effects have a crucial role for ensuring the stability of the system. However, to make this appear clearly, we develop in this section a purely three body model, ignoring thus tidal effect which will be reintroduced afterwards.

3.1. NUMERICAL STUDY

In order to investigate the dynamics of this triple system, we carried out a numerical integration of the averaged system, taking as input the different orbital fit solutions from [7]. The theoretical background is developed in [3].

Figure 2 shows an example of temporal evolution. The first obvious conclusion is that the orbit of the binary is much more affected by perturbations than that of the tertiary. Surprisingly, the binary orbit appears to become regularly highly eccentric, while its inclination drops sharply at the same time the eccentricity becomes large. Conversely, the orbit of the tertiary appears much more stable. This general behavior was recorded for all input solutions.

3.2. INTERPRETATION

The recorded dynamical behavior may be understood when comparing the angular momenta of the orbits. It is worth noticing that the angular momentum of the tertiary orbit is significantly larger than that of the orbit of the binary (~ 5 times more). This ratio is not subject to evolution during the motion. Therefore, the angular momentum of the tertiary is forced to remain permanently close to the global $\vec{C}$, showing that i) the eccentricity of the tertiary should not vary drastically ii) its orbital plane should remain roughly perpendicular to $\vec{C}$. This is obviously confirmed by the numerical integration.

Conversely, the orbit of the binary is expected to be the subject of more drastic changes. The behavior reported above may be described in the frame of the *Kozai mechanism* [16, 2]. This mechanism applies in the Solar System for comets with originally highly inclined orbits with respect of the ecliptic. Under the effect of secular planetary perturbations, the orbit of the comet evolves, but the z-component of the orbital angular momentum of the comet (hereafter J_z) is expected to remain constant. This causes an orbit initially highly inclined and weakly eccentric to potentially evolve to weakly inclined but highly eccentric. This occurs indeed periodically with the precession of the argument of perihelion.

A similar mechanism is at work here on the orbit of the binary. Since the orbit of the tertiary is close to the (OXY) plane perpendicular to the global angular momentum, it is easy to show by rotational invariance of the hamiltonian equations (see [3]) that J_z for the binary orbit should be roughly constant, which is the context of the Kozai mechanism.

4. Tidal effects

4.1. THE NEED FOR CONSIDERING TIDAL EFFECTS

The three body calculations presented in the previous section did not take into account any tidal effect between the components of the clase binary of the TY CrA system. However, this cannot be avoided. It is well known that binaries with orbital periods less than a cut-off period are expected to be circularized that way. In any case, the orbital period of the close binary of the TY CrA system is far less than the cut-off value [14, 20, 19].

Independently from this, Fig. 2 shows obviously that the three-body model of the previous section cannot be satisfactory: the eccentricity of the binary is expected to sometimes almost reach 1 (the exact peak value was about 0.995). Remembering that the semi-major axis is constant, one sees that the periastron distance between the two primaries must become very small whenever the eccentricity is high, showing that the binary should in fact have already coalesced into a single star unless tidal effects prevent the orbit from evolving to high eccentricity values.

4.2. VARIOUS TIDAL MECHANISM

Several mechanisms generate tidal effects in close binaries. They all tend to synchronize the rotation of both components with the orbital motion and to circularize the orbit. In the case when the rotation axes would not be perpendicular to the orbital plane, the tidal effects also act to align both orbital and rotational angular momenta. This is valid for any tidal mechanism [10]. The efficiency of the different mechanism may be compared evaluating characteristic circularization (t_{circ}) and synchronization (t_{sync}) times for each of them.

The first mechanism invoked is the *equilibrium tide*: due to tidal attraction from its companion, each component is distorted. The analysis of this effect was achieved by Alexander [1], Press et al. (1975), Kopal [15] and Hut [11, 12], in the so-called weak-friction case.

Another tidal mechanism inducing synchronization and circularization known as *dynamical tides* was analyzed by Zahn [23, 24]. This occurs when the non-adiabatic oscillations driven on one component by the perturbing action of its companion are damped by radiative dissipation. This leads to a torque applied to the star, thus coupling orbital motion and rotation.

A final mechanism was discovered by Tassoul & Tassoul ([22] and Refs. therein). They showed that tidally driven meridional currents within each component act for circularization and synchronization. They also gave characteristic times.

4.3. APPLICATION TO TY CrA

Comparing the efficiency of the different tidal mechanism described above means evaluating and comparing the values of the corresponding characteristic times. The formulas giving these times are given in the corresponding references and in [3]. One needs thus values for the various constants appearing in these expressions, once applied to the peculiar case of the close binary of the TY CrA system. Among them we first have the apsidal constants k_j for each component (see [24]).

We thus tabulated these quantities over the lifetime of both components of the binary. More precisely, for each component at a given age, we used the stellar models by Siess et al. [8] to obtain them. The result is illustrated on Fig. 3, where k_2 is plotted as a function of time for both components. Schematically, for each star, k_2 remains high as long as the star is pre main-sequence, and drops as soon as a radiative core grows. The temporal position of the two possible ages t_{PMS} and t_{MS} mentioned above is indicated

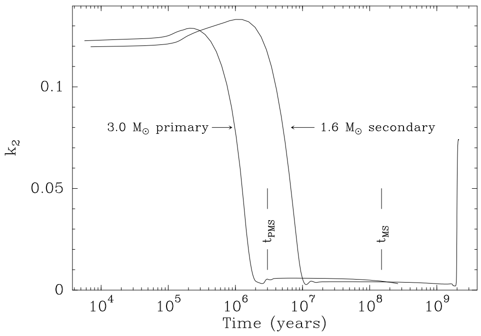

Figure 3. Evolution of the first apsidal constant k_2 as a function of time for the components of the TY CrA central binary, assuming solar metallicity. The temporal location of $t_{\rm MS}$ and $t_{\rm PMS}$ for solar metallicity is indicated.

TABLE 1. Values of characteristic times for the different tidal mechanisms invoked.

Mechanism ($t_{\rm PMS}$: $3\,10^6$ yr)	$t_{\rm circ}$ (yrs)	$(t_{\rm sync})_1$ (yrs)	$(t_{\rm sync})_2$ (yrs)
Equilibrium tide with turbulence [21]	$2.488\,10^{10}$	$1.458\,10^{11}$	$1.175\,10^{8}$
Dynamical tide [24]	$2.488\,10^{10}$	$6.246\,10^{7}$	$5.246\,10^{8}$
Meridional circulation [22]	$2.028\,10^{6}$	$2.765\,10^{4}$	$1.993\,10^{4}$
Mechanism ($t_{\rm MS}$: $1.5\,10^8$ yr)	$t_{\rm circ}$ (yrs)	$(t_{\rm sync})_1$ (yrs)	$(t_{\rm sync})_2$ (yrs)
Equilibrium tide with turbulence [21]	$3.846\,10^{12}$	$1.219\,10^{10}$	$1.193\,10^{11}$
Dynamical tide [24]	$1.134\,10^{10}$	$4.129\,10^{7}$	$1.926\,10^{9}$
Meridional circulation [22]	$3.903\,10^{6}$	$2.001\,10^{4}$	$4.762\,10^{4}$

(for solar metallicity). We clearly see on the plot the difference between these two models. At $t_{\rm MS}$, both stars are basically on the main sequence, while this is only the case for the $3.0\,M_\odot$ primary at $t_{\rm PMS}$.

Given these results, we thus computed the various characteristic times for all the tidal effects, for both ages. The results are listed in Table 1. We may note that in any case, the most powerful effect is the meridional circulation model by Tassoul & Tassoul [22].

5. Dynamics of the TY CrA system with tidal effects

We carried out a numerical integration of the TY CrA system dynamics, adding the tidal effects to the basic three-body model described above, and taking now into account the evolution of the rotation of the components of the binary. We here only retained the strongest effect, i.e., the meridional circulation model by by Tassoul & Tassoul [22].

If the binary was alone, there would be no reason for both rotation axes not to be perpendicular to the orbital plane. However, the presence of the tertiary component and the non-coplanarity of the orbits may change this simple picture. It is therefore legal to consider that the rotation axes might

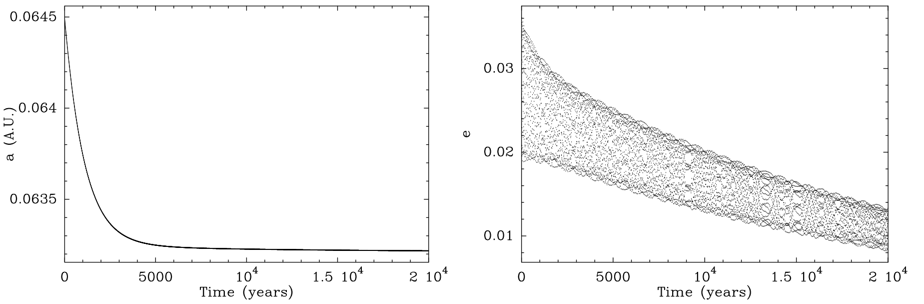

Figure 4. Evolution of the semi-major axis a and eccentricity e of the binary over 20 000 yrs under the effects of dissipative tidal effects, at t_{PMS}. An equilibrium value is reached within ∼6 000 yrs.

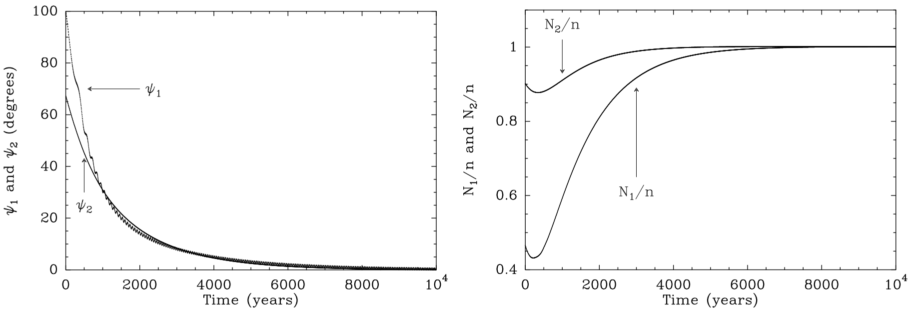

Figure 5. Evolution of the tilt angles ψ_1 and ψ_2 characterizing the alignment of the rotation axes of the binary, under the effects of dissipative tidal effects, at t_{PMS}. We note a quick alignment/synchronization process.

not be aligned.

For each age (t_{PMS} or t_{MS}), several runs with different initial conditions were carried out; they all revealed similar behaviors, showing that the dynamics depends only weakly on the initial conditions. However, significant differences appeared between the behavior recorded for the two models. We present now two typical runs, one for each age.

5.1. THE TYPICAL BEHAVIOR

The dissipative tidal effects act on circularization, synchronization and alignment of the rotation axes with the orbital angular momentum within the close binary. However, such an equilibrium holds theoretically for an isolated binary. It is however worth noticing that a similar equilibrium may be reached for the 3-body TY CrA system. We first show results at t_{PMS}. Figure 4 shows the evolution of the semi-major axis a and the eccentricity e of the close binary. We see that after a smooth decrease, we reach a sta-

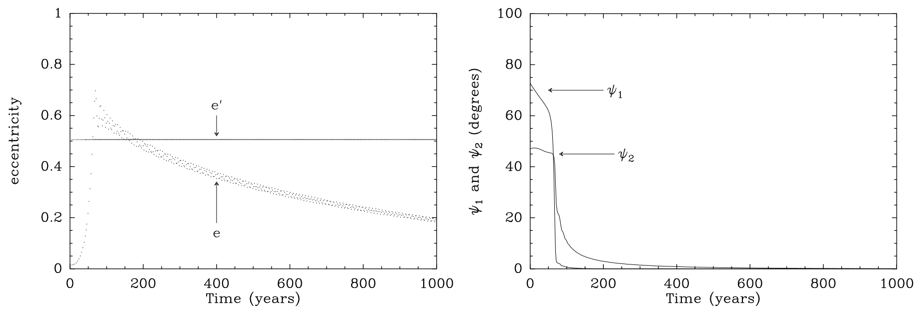

Figure 6. Evolution of the eccentricities of both orbits under the effects of dissipative tidal effects, and of the tilt angles ψ_i's for the components of the binary, at $t_{\rm MS}$. We note a gradual circularization of the binary following a strong increase of the eccentricity. We also see that the alignment process is much more rapid than at $t_{\rm PMS}$.

ble equilibrium value. Contrary to Fig. 2, the eccentricity remains small, and slowly decreases; thanks to the tidal effects, no Kozai-like behavior is present. The stability of the TY CrA system appears actually ensured by tidal effects !

Figure 5 describes the evolution of the rotation of the components of the binary. It is convenient here to display the evolution of the tilt angles ψ_i's $(i = 1, 2)$ between the rotation axes of each star and the orbital angular momentum of the binary. We see that the rotation axes quickly move to align with the orbital angular momentum of the binary.

Figure 5 also shows the evolution of the rotation velocities N_i of the stars, once divided by the mean orbital motion n (thus, 1 means synchronism). We see that both stars are synchronized within ~6 000 yrs.

The behavior described here is characteristic for a tidally interacting binary. Moreover, as expected (see Table 1), the synchronization is achieved earlier than the circularization. It was shown by Hut [11] that this fact is valid for any binary for which the orbital angular momentum largely overcomes the rotational one. However, it is a remarkable fact that this also applies to the TY CrA system, despite the presence of the third companion.

We come now to present results of a similar run at $t_{\rm MS}$. Figure 6 shows the temporal evolution of the eccentricity e of the binary, and of the tilt angles ψ_i's $(i = 1, 2)$. They must be compared to Figs. 4 and 5.

Here, we note a strong initial increase of the eccentricity. Afterwards, it slowly decreases to zero. The sudden increase of the eccentricity is accompanied by a very rapid synchronization and a quick alignment of the axes, which take place much earlier than at $t_{\rm PMS}$. This may be explained as follows: Thanks to smaller tidal coupling at $t_{\rm MS}$ (see Fig. 3 or Table 1), the 3-body dynamics is strong enough to initiate a full Kozai cycle, making the periastron distance between both components decrease. This leads to a

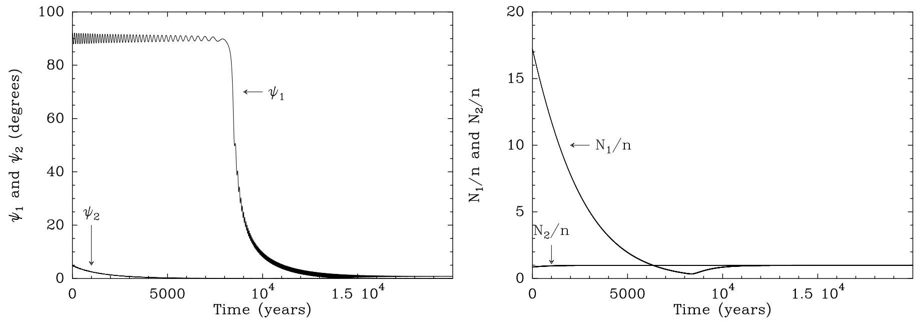

Figure 7. Evolution of the tilt angles ψ_1 and ψ_2 and rotation velocties (N_1/n, N_2/n) over 20 000 yrs, under the effects of dissipative tidal effects, at $t_{\rm PMS}$ with an initially pole-on primary. The $3.0\,M_\odot$ primary remains quite a long time at $\psi_1 = 90°$, while the $1.6\,M_\odot$ secondary is quickly aligned. This remains true even when the initial value of ψ_2 is high. We see that the $3.0\,M_\odot$ primary is initially highly super-synchronous, despite a very low $v\sin(i)$. This is due to the pole-on orientation of its rotation axis.

strong increase of the tidal effects. The consequence is a rapid synchronization of the binary. Finally, the equilibrium is reached more quickly than at $t_{\rm PMS}$, *thanks* to the 3-body dynamics, and *although* the tidal effects are globally smaller (Table 1).

5.2. A PECULIAR BEHAVIOR

Testing various initial configurations of the rotation axes reveals that the behavior described above is quite generic. We come now to describe the results of a specific run which might give clues for understanding the sub-synchronosm problem of the primary. We first recall that in all our runs, once the orientation of the rotations axes are chosen, the initial rotation velocities is fixed to fit the observed present $v\sin(i)$'s of the stars ($8\,{\rm km\,s^{-1}}$ for the primary and 35 km for the secondary [4, 6, 7]). In the following run, the initial orientation of the $3.0\,M_\odot$ primary was nearly pole-on; thus, its initial rotation velocity was high (super-synchronous), contrary to all other possible orientations.

Figure 7 shows the results of this run at $t_{\rm PMS}$: we first note that the same equilibrium as before is obtained, but somewhat later, the tilt angle ψ_1 of the $3.0\,M_\odot$ primary remaining quite a long time close to 90° before dropping to zero.

It thus seems that a marginally stable equilibrium $\psi_1 = 90°$ exists when the rotation of the primary is high enough. $\psi_1 = 90°$ means that the rotation axis of the $3.0\,M_\odot$ primary lies in the orbital plane of the binary. Such a marginal equilibrium might apply for the today situation of the TY CrA system, and thus explain why the binary does not appear synchronized. As the binary is an eclipsing one, $\psi_1 = 90°$ is compatible with a pole-on orientation of the rotation axis of the primary with respect to the line of sight. Before concluding anything, we must examine the origin of this marginal equilibrium. First, we may note that it is not observed at $t_{\rm MS}$ (see [3]). In that case, the tilt angles ψ_i's drop to zero much more quickly than at $t_{\rm PMS}$, within less than 500 yrs.

5.3. INTERPRETATION

In order to understand this peculiar behavior, one has to describe the variations of ψ_1. The details of the calculations are given in [3]. Schematically, to lowest order, we may write:

$$\frac{{\rm d}\psi_1}{{\rm dt}} = \left(\frac{{\rm d}\psi_1}{{\rm dt}}\right)_{\rm tidal} - \sin\eta_1 \sin i \left(\frac{{\rm d}\Omega}{{\rm dt}}\right)_{3-{\rm body}}, \tag{1}$$

where η_1 is a rotation angle around the angular momentum of the binary, defining (with ψ_1) the direction of the rotation axis of the primary. Here $({\rm d}\psi_1/{\rm dt})_{\rm tidal}$ stands for the tidal contribution, while the second term is explicitely due to the three body interaction, by means of the precession velocity $({\rm d}\Omega/{\rm dt})_{3-{\rm body}}$ of the orbital plane of the binary. This velocity is roughly constant (see [3]). Once applied to the TY CrA system, it appears that that $({\rm d}\psi_1/{\rm dt})_{\rm tidal} \ll ({\rm d}\Omega/{\rm dt})_{3-{\rm body}}$. Hence, by virtue of Eq. (1), a stable point for ψ_1 implies $\sin\eta_1 \simeq 0$. It is then easy to derive that as soon as $N_1/n \gtrsim 0.5$, the only stable point is closer to $90°$ than $0.2°$. In fact, contrary to an isolated binary, the equilibrium is controlled by the 3-body term of Eq. (1) which dominates the tidal term.

6. Long-term evolution of the TY CrA system

The theory described in the previous section appears indeed well suited to explain the apparent non-synchronism of the central binary of TY CrA. The 3-body dynamics enhances a pseudo-equilibrium constraining the rotation axis of the primary to the orbital plane of the binary. However, we see from Fig. 7 that the duration of this phase should not exceed $\sim 10\,000$ years. Even if the age of the TY CrA system is unknown, this is a very short duration. In fact, if we assume the values given by Table 1 for the characteristic times of the meridional circulation model by Tassoul & Tassoul [22], there is no way to explain a non-synchronism of the binary after at most 10^5 years. One has then to admit than these constants are significantly smaller in the present case. This might be justified by the fact that all these quantities highly depend on the internal viscosity of the star, which is in any case very poorly known.

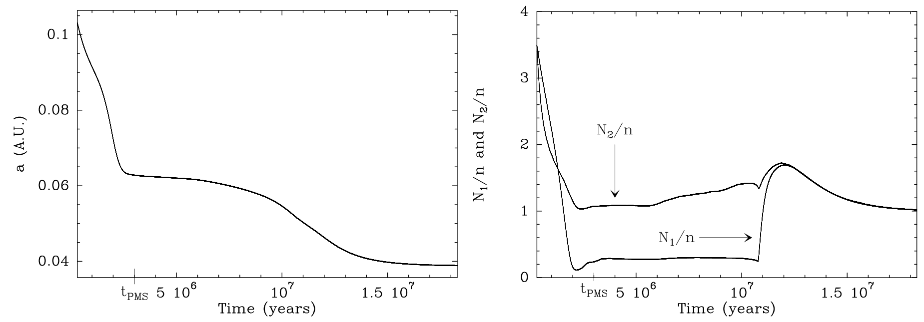

Figure 8. Long-term evolution of the semi-major axis and rotation velocities of the central binary under tidal effects, with reduced Tassoul constants, for $1.8\,10^7$ yrs. The semi-major axis regularly decreases. The value at $t_{\rm PMS}$ corresponds roughly to the present value. Both stars are initially supersynchronous, but a gradual slow-down process is detected. The stars quicly synchronize after 10^7 yrs. Note that the primary remains significantly subsynchronous for a long time.

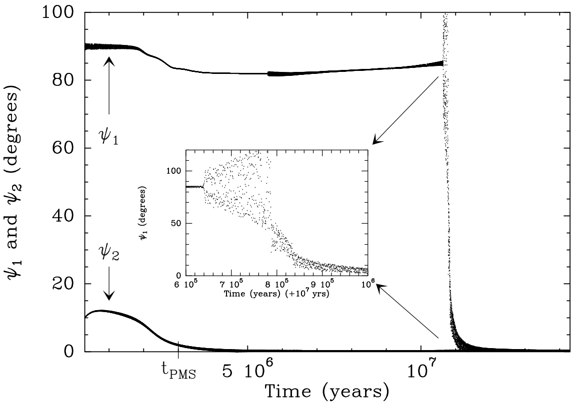

Figure 9. Same as Fig. 8, but for the tilt angles ψ_i $(i = 1, 2)$. The rotation axis of the primary remains constrained to the precessing orbital plane of the binary for $\sim 1.1\,10^7$ yrs. The central inner plot is an enlargement of the transition epoch for ψ_1 between $\sim 90°$ and $\sim 0°$. The alignment is achieved within a few 10^5 yrs.

In order to test this theory, we decided to investigate the dynamics of TY CrA over a longer time-scale (a few 10^7 yrs), with significantly reduced Tassoul constants. Compared to the previous runs, we had to take into account the fact that the internal structure of the stars may evolve over such a time-scale. This is why we decided to introduce the evolution of all concerned quantities at every time during the integration, using the models by Siess et al. [8]. Our aim was mainly to investigate the long-term stability of the marginal $\psi_1 = 90°$ equilibrium, so that we gave this initial value to ψ_1.

Figures 8 and 9 show the result of an integration with such initial conditions for $1.4\,10^7$ yrs. A reduction factor of 400 was applied to the Tassoul effect, and the initial semi-major axis is 1.6 times the present value. We see that the binary gradually synchonizes over this time-scale, while the rotation axes remain constrained to the precessing orbital plane of the binary for a few million years before aligning. This behavior is similar to that of Fig. 7, but the time-scale is much longer ($\sim 10^7$ yrs). This time-scale must not be surprising, since according to Fig. 3, 10^7 yrs corresponds to the time when both components of the primary have roughly reached the main sequence, i.e., when the tidal effects are weaker.

7. Conclusion

The long-term run shows that the survival of the marginal equilibrium characterized by $\psi_1 \simeq 90°$ is able to last several 10^6 yrs long, provided the tidal effects are not too strong. If we assume that the present age of TY CrA is $t_{\rm PMS} \simeq 3\,10^6$ yrs, this situation could still be valid today. However, it is unrealistic to imagin that the marginal equilibrium could still survive at $t_{\rm MS}$.

We may therefore suggest that i) the present age of TY CrA is probably $t_{\rm PMS}$ rather than $t_{\rm MS}$ and ii) the present status of the rotation axes of the binary corresponds to the marginal equilibrium. This requires the Tassoul & Tassoul tidal mechanism to be in the case of TY CrA significantly smaller than usually. We think that this is not in itself a very strong constraint, In any case, the stability of the system first appears ensured by tidal effects.

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